[gex114] Linear inhomogeneous ODE: undetermined parameters I

Consider the linear, inhomogeneous 2nd-order ODE,

$$y'' + 4y = 8\sin(2x).$$

(a) Examine the characteristic polynomial of the homogeneous ODE and identify if the solution is of case #1 (real roots), case #2 (complex roots), or case #3 (repeated roots) [gam8].

(b) Use the DSolve command of Mathematica to find the complementary solution $Y_c(x)$, which is the general solution of the homogeneous ODE and features two integration constants.

(c) Use the DSolve command to find the general solution y(x) of the inhomogeneous ODE.

(d) Extract the particular solution $Y_{\rm p}(x) = y(x) - Y_{\rm c}(x)$ from the results of parts (a) and (b).

(e) Determine $Y_p(x)$ by the method of undetermined constant parameters [gam8] and reconcile the difference with the expression found in part (c).

Solution: