[gex112] From polar to rectangular coordinates and back: Jacobians

Consider the transformation from polar coordinates to rectangular coordinates,

$$\mathcal{T}: \; x = \rho\cos\phi, \quad y = \rho\sin\phi,$$

and the inverse transformation,

$$\mathcal{T}^{-1}: \; \rho = \sqrt{x^2 + y^2}, \quad \phi = \arctan\frac{y}{x}.$$

Remember that these transformations are nonlinear and orthogonal.
(a) Use the Mathematica command D (for partial derivative) to calculate the Jacobian matrix $\mathbf{J}$ of the transformation $\mathcal{T}$.
(b) Use the command Inverse to calculate the inverse $\mathbf{J}^{-1}$ of this Jacobian.
(c) Now calculate the Jacobian $\bar{\mathbf{J}}$ of the inverse transformation $\mathcal{T}^{-1}$.
(d) Show that the $\bar{\mathbf{J}} = \mathbf{J}^{-1}$.
(e) Express the elements of both $\mathbf{J}$ and $\mathbf{J}^{-1}$ as functions of x, y and as functions of ρ, ϕ.
(f) Demonstrate that $\mathbf{J} \cdot \mathbf{J}^{-1} = \mathbf{I}$ holds in both representations of the Jacobians.

Solution: