

[gex111] Linearly damped harmonic oscillator: general solution

The linear, homogeneous, 2nd-order ODE,

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0,$$

describes the time evolution of the linearly damped harmonic oscillator [gam8].

(a) Determine the general solution for initial conditions $x(0) = x_0$, $\dot{x}(0) = \dot{x}_0$ separately for the cases of (i) overdamped motion ($\beta^2 > \omega_0^2$), (ii) underdamped motion ($\beta^2 < \omega_0^2$), and (iii) critically damped motion ($\beta^2 = \omega_0^2$). Use the Mathematica command `DSolve` for this purpose.

(b) Demonstrate that the solution found via Mathematica is equivalent to the solution stated in [gam8] for all three cases.

(c) Show that the solutions found by Mathematica for the cases (i) and (ii) become the solution found for the case (ii) in the limit $\omega_0 \rightarrow \beta$. Use the Mathematica command `Limit` for this purpose.

Solution: