[gex109] Second-order ODE: reduction to first order ODE II

Consider the 2^{nd}-order ODE for the function $y(x)$,

$$yy'' + y'^2 + 1 = 0,$$

which is amenable to a reduction into a 1^{st}-order ODE for the variable $z(y) = y'$.
(a) Solve the original 2^{nd}-order ODE via the DSolve command of Mathematica and show that the solution can be rendered as follows:

$$(x + b)^2 + y^2 = a^2,$$

where a, b are conveniently chosen integration constants.
(b) State and solve the 1^{st}-order ODE for $z(y)$ via the DSolve command.
(c) The solution $z(y)$ with $z = dy/dx$ substituted is, effectively, a1^{st}-order ODE for $y(x)$. Use the DSolve command for that and compare the resulting $y(x)$ with the result of part (a).

Solution: