[gex107] Coupled first-order ODEs: Rössler band strange attractor

The set of 1^{st}-order ODEs,

$$\dot{x} = -y - z, \quad \dot{y} = x + \frac{1}{5}\, y, \quad \dot{z} = \frac{1}{5} + z(x - 5.7). \tag{1}$$

featuring one seemingly innocuous nonlinearity is known to include as an invariant structure the well known strange attractor, named Rössler band after its discoverer. This invariant structure is of fractional dimensionality. It has infinite area, but zero volume. It is a ribbon being continually stretched, bent, and squeezed as described in [gam3].

(a) Use the Mathematica commands NDSolve and ParametricPlot3D to generate and visualize a trajectory from an initial condition at or near $(0, 0, 0)$. Such a trajectory will be be quickly attracted to the Rössler band and not leave it again even though its exact course is subject to exponentially fast error propagation.

(b) The ODEs (1) also feature two fixed point, neither of which is an attractor. Determine the locations of the two fixed points. Then determine the three eigenvalues of each fixed point, showing they are both saddle points.

Solution: