[gex106] Coupled first-order ODEs: fixed point and limit cycle

Consider a flow in the (x, y)-plane governed by the coupled 1st-order ODEs,

$$\dot{x} = x\left(1 - \sqrt{x^2 + y^2}\right) - y, \quad \dot{y} = y\left(\tfrac{1}{2} - \sqrt{x^2 + y^2}\right) + x.$$

(a) Investigate this flow of this system in th (H, P)-plane for $-1.5 < x, y < 1.5$ by employing the StreamPlot command of Mathematica. It reveals the presence of fixed point and a limit cycle.
(b) Determine the location and the type of the fixed point.
(c) Superimpose on the StreamPlot a graph of the limit cycle determined via numerical integration of the ODEs with judiciously chosen initial conditions. Use the commands NDSolve and ParametricPlot for this purpose.

Solution: