[gex105] Coupled first-order ODEs: fixed points and flow dynamics II

The coexistence of two species of host and parasite populations is modeled by the following coupled 1^{st}-order ODEs for their instantaneous numbers $H(t)$ and $P(t)$:

$$\dot{H} = (1 - P)H, \qquad \dot{P} = P\left(1 - \frac{2P}{1 + H}\right).$$

The point of this exercise (and that of [gex104]) is to demonstrate that a great deal can be learned about the solutions of ODEs without actually producing analytical or numerical integrations.
(a) Investigate the flow of this system in the (H, P)-plane for $0 < H, P < 1.5$ by employing the StreamPlot command of Mathematica, which reveals the presence and (at least) approximate location of three fixed point.
(b) Use the Solve command to identify the exact location of the three fixed points.
(c) Determine the type of each fixed point by analyzing the eigenvalues of its Jacobian matrix. Use the Mathematica command Eigenvalues for this purpose.

Solution: