

[gex104] Coupled first-order ODEs: fixed points and flow dynamics I

A population F of foxes feeds on a population H of hares. The birth rate of foxes is proportional to the fox population and to the amount of food available. Foxes die at a rate proportional to the fox population. Hares die primarily through encounters with foxes and are born at a rate proportional to the hare population:

$$\dot{H} = aH - bHF, \quad \dot{F} = cHF - dF,$$

where a, b, c, d are positive constants and $H \geq 0, F \geq 0$ is assumed.

- (a) Find all fixed points in the (H, F) -plane and determine their types. Use the `StreamPlot` command of Mathematica to visualize the flow in the (H, F) -plane.
- (b) Interpret the population dynamics on the basis of the flow in the (H, F) -plane.
- (c) If the population of hares is suddenly cut in half by an epidemic from which the remaining hares are immune, discuss the different effects this can have on the system depending on the size of both populations at the time of the epidemic.

Solution: