[gex103] Plane pendulum with attenuation: fixed points and phase flow

Consider the equation of motion of the plane pendulum,

$$\ddot{\theta} + 2\beta\dot{\theta} + \omega_0^2 \sin\theta = 0,$$

where θ is the angular coordinate, $\omega_0 = \sqrt{g/L}$ the characteristic frequency and β the parameter controlling attenuation.
(a) Convert this 2^{nd}-order ODE into a pair of 1^{st}-order ODEs for $x(t) = \theta$ and $y(t) = \dot{\theta}$.
(b) Identify the locations in the two fixed points in the (x, y)-plane.
(c) Determine the nature of the two fixed points for (i) zero damping ($\beta = 0$), (ii) weak damping ($\beta < \omega_0$), (iii) critical damping ($\beta = \omega_0$), and (iv) strong damping ($\beta > \omega_0$).
(d) Use the Mathematica StreamPlot command to graphically present the phase flow near the fixed point associated with $\theta = 0$ for the cases (i)-(iv). Adjust the style and range of your graph to enhance the visibility of the differences between the four cases.

Solution: