Special Functions [gmd4]

In a sense, all functions are special, namely in applications where they prove useful. Some functions are more widely used than others. They have familiar properties – nothing special in the sense of 'haute cuisine.'

The emergence of a new function on the scene is a combination of discovery and invention. Sometimes it happens in the course of the analysis of a particular physics problem.

Functions first specified and described in a particular context are likely to find additional applications elsewhere, e.g. in situations of shared symmetry.

What follows in this module is not meant to be a complete or systematic compilation of special functions. It is to be understood as a (somewhat disorderly) toolbox of a working physicist.

Elementary functions:

The most elementary functions assign a real number $y = f(x)$ (dependent variable) to a real number x (independent variable) from a given set, subject to certain restrictions regarding continuity.

A list of functions that might be called elementary:

– polynomials,
– rational functions,
– algebraic functions,
– exponential functions,
– trigonometric functions,
– hyperbolic functions,
– logarithmic functions,
– inverse trigonometric and hyperbolic functions.

Interpreted as real functions of real variables, they are continuous and differentiable. Some have divergences, others are multiple-valued.

Attributes of these elementary functions and relations between them are more interesting and revealing in the context of *complex analysis* [gmd7].

Special functions:

Less familiar functions are being discussed in groups that are more closely related according to somewhat vague criteria. The ordering is somewhat arbitrary as well.

Some special functions are defined as integrals of more elementary functions, others are defined as solutions of differential equations.

- [gmd4A] Gamma function and related functions:
  - Beta function,
  - incomplete Gamma function,
  - Digamma function,
  - Polylogarithmic functions,
  - Polygamma functions,
  - Zeta function,
- [gmd4B] Error function and related functions:
  - Fresnel integrals,
  - exponential integral,
  - logarithmic integral,
  - sine integral and cosine integral.
- [gmd4C] Elliptic integrals and elliptic functions:
  - incomplete elliptic integrals of the first, second, and third kind,
  - complete elliptic integrals of the first and second kind,
  - Jacobi elliptic functions.
- [gmd4D] Legendre functions:
  - Legendre polynomials,
  - Legendre functions,
  - associated Legendre functions.
- [gmd4E] Spherical harmonics:
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