[lex99] Fresnel equation for TM wave

Consider a TM plane wave refracted and reflected at the interface between dielectrics as shown. The boundary conditions for the normal and tangential fields produce, as shown in [lln17], the two relations for the electric-field amplitudes,
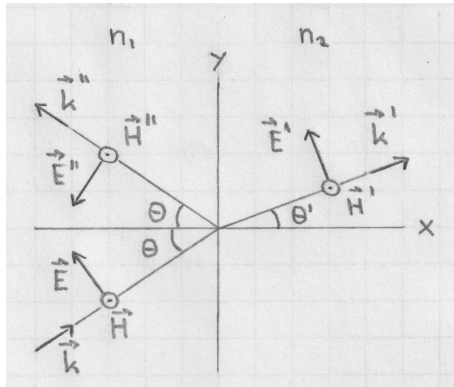
$$\frac{n_1}{\mu_1}(E_0 + E_0'') = \frac{n_2}{\mu_2}E_0', \quad (E_0 - E_0'')\cos\theta = E_0'\cos\theta'.$$

(a) Show that these relations determine the amplitude ratios,

$$\frac{E_0'}{E_0} = \frac{2\mu_2 n_1 \cos\theta}{\mu_1 n_2 \cos\theta + \mu_2 n_1 \cos\theta'}, \quad \frac{E_0''}{E_0} = \frac{\mu_1 n_2 \cos\theta - \mu_2 n_1 \cos\theta'}{\mu_1 n_2 \cos\theta + \mu_2 n_1 \cos\theta'}.$$

(b) Infer from the second relation, for the situations with $\mu_1 = \mu_2 = \mu_0$, the simplified version (Fresnel equation),

$$\frac{E_0''}{E_0} = \frac{\tan(\theta - \theta')}{\tan(\theta + \theta')}.$$

Solution: