[lex97] Anti-reflection coating

Incident light is partially reflected from the surface of a dielectric material with index of refraction n_2. By coating the surface with a dielectric material of specific thickness a and specific index of refraction n_1, the surface can be made effectively nonreflective. Here we consider the case of normal incidence as illustrated.
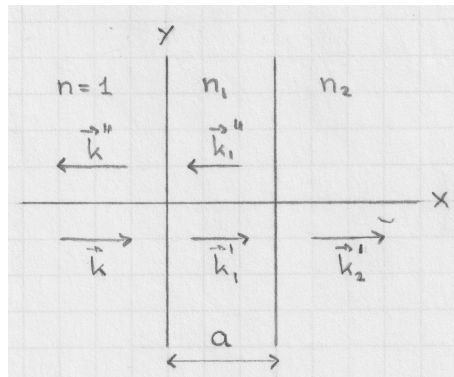
In [lln17] we have established, from the surface and interface boundary conditions for tangential electric and magnetic fields, the following phase relations for the electric field of the five waves:

$$E_0 = E_1' + E_1'' = n_1(E_1' - E_1''), \quad E_1' e^{\imath k_1 a} + E_1'' e^{-\imath k_1 a} = \frac{n_1}{n_2}\left[E_1' e^{\imath k_1 a} - E_1'' e^{-\imath k_1 a} \right], \quad k_1 = n_1 \frac{\omega}{c}.$$

(a) Infer from these relations the following condition for anti-reflection coating:

$$\frac{n_2}{n_1} = \frac{\imath\, n_1 \sin(k_1 a) + \cos(k_1 a)}{n_1 \cos(k_1 a) + \imath \sin(k_1 a)}.$$

(b) Physically relvant solutions for n_1 and a require that the right hand-side is real, which is the case if (i) $\sin(k_1 a) = 0$ or (ii) $\cos(k_1 a) = 0$. Discuss the nature of both solutions. Only one of the serves the intended purpose.

Solution: