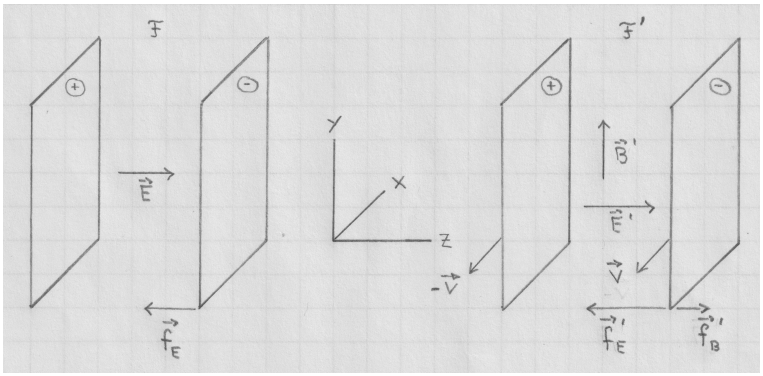
[lex95] Fields between moving capacitor plates I

Two oppositely charged conducting plates are placed in a coordinate system as shown. In the rest frame $\mathcal{F}$ of the plates, there is a uniform electric field $\mathbf{E} = E_0\,\hat{\mathbf{k}}$ between the plates. The Lorentz transformation predicts that in the frame $\mathcal{F}'$, which moves with velocity $\mathbf{v} = v\,\hat{\mathbf{i}}$ relative to $\mathcal{F}$, the electric field is stronger and there is also a magnetic field:

$$\mathbf{E}' = \gamma E_0\,\hat{\mathbf{k}}, \quad \mathbf{B}' = \frac{\gamma v}{c^2}\,E_0\,\hat{\mathbf{j}}, \quad \gamma \doteq \frac{1}{\sqrt{1 - v^2/c^2}}.$$

(a) Explain the change in electric field as due to the contracted plate areas when viewed from $\mathcal{F}'$.
(b) Explain the appearance of the magnetic field as an effect of the moving charges on the plates (with contracted areas) when viewed from $\mathcal{F}'$.
(c) Show that the net force per area between the plates, which is purely electric when viewed from $\mathcal{F}'$, but a combination of electric and magnetic when viewed from $\mathcal{F}'$, is invariant under a change of reference frame, as demanded by the principle of relativity.



Solution: