[lex89] Energy delivery into current-carrying wire.

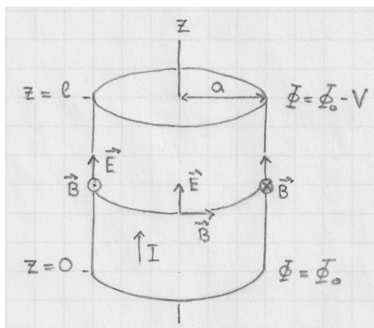
A wire of radius a carrying a steady current I dissipates energy at a constant rate. That energy comes from the power supply (voltage source) to which the wire is connected. How is it distributed along the wire to be converted into thermal energy? An application of the Poynting theorem [lln15],

$$-\frac{\partial u}{\partial t} = \nabla \cdot \mathbf{S} + \mathbf{E} \cdot \mathbf{J}, \quad \text{where} \quad \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B},$$

sheds some light on this question. Here u [$\mathrm{Jm^{-3}}$] is the electromagnetic energy density, $\mathbf{J}$ [$\mathrm{Am^{-2}}$] is the current density, and the Poynting vector $\mathbf{S}$ [$\mathrm{Jm^{-2}s^{-1}}$] represents the electromagnetic energy current density. We assume that inside the wire segment of length l shown, $\mathbf{E}$ and $\mathbf{J}$ are uniform and pointing in z-direction. The potential drop across the wire segment is V.
(a) Express the two fields $\mathbf{E}$ and $\mathbf{B}$ at a point on the surface of the wire. Use cylindrical coordinates.
(b) Determine magnitude and direction of the resulting Poynting vector $\mathbf{S}$ at the same point.
(c) Calculate the energy flux,

$$\Phi_S \doteq \oint \mathbf{S} \cdot d\mathbf{a} \quad [\mathrm{Js^{-1}}],$$

flowing through the surface of the wire segment.
(d) Compare the result with the power $P = VI$ dissipated inside the wire segment. Draw your own conclusions regarding the energy flow along the wire.



Solution: