[lex84] Vector and scalar potentials of spherical wave

The scalar potential $\Phi(\mathbf{x},t)$ and the vector potential $\mathbf{A}(\mathbf{x},t)$ in a region of space without sources (no charges and no currents) must satisfy the wave equation,

$$\nabla^2\Phi = \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}, \quad \nabla^2\mathbf{A} = \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}, \tag{1}$$

if the Lorenz gauge is chosen.
(a) Show that the ansatz,

$$\mathbf{A}(\mathbf{x},t) = \psi(r,t)\,\hat{\mathbf{k}}, \quad \psi(r,t) = \frac{C}{r}\,e^{\imath(kr-\omega t)}, \tag{2}$$

invoked in [lln15] for a spherical wave, where C is an arbitrary amplitude, does indeed satisfy the wave equation.
(b) Use the Lorenz gauge condition,

$$\nabla\cdot\mathbf{A} = -\mu_0\epsilon_0\,\frac{\partial\Phi}{\partial t},$$

to infer the scalar potential $\Phi(\mathbf{x},t)$ from the vector potential $\mathbf{A}(\mathbf{x},t)$.

Solution: