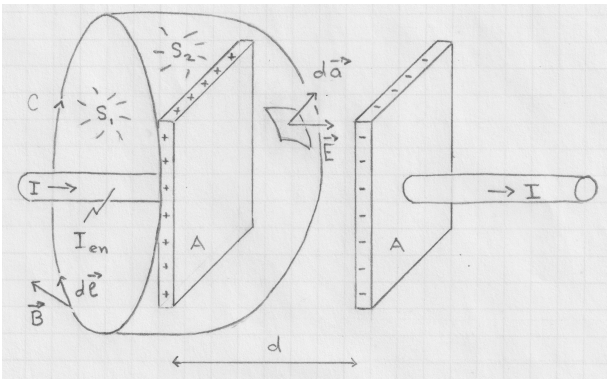
[lex82] Conduction current versus displacement current I

In the setup shown a parallel-plate capacitor with plates of area A a distance d apart is being charged by a conduction current I in on the left and out on the right via connecting wires. No conduction current flows between the plates, but a uniform electric field $\mathbf{E}$ is being built up. Ampère's law in integral form reads

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_S (\mathbf{J} + \mathbf{J}_D) \cdot d\mathbf{a}, \quad \mathbf{J}_D = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

The loop C is perimeter to surfaces S_1 and S_2. Show that the right-hand side of Ampère's law is identical for both surfaces, even though for surface S_1 there is only conduction current density $\mathbf{J}$ and for surface S_2 only displacement current density $\mathbf{J}_D$.



Solution: