[lex8] Electric field of parabolic line charge

Place a parabolic line charge of infinite length and with uniform line charge density λ into the xy-plane such that $y = ax^2$ with $a > 0$.
(a) Show that the electric field at position y_0 on the y-axis is given by the integral expression,

$$E_y = 2k\lambda \int_0^\infty dx \frac{\sqrt{1+4a^2x^2}(y_0 - ax^2)}{[x^2 + (y_0 - ax^2)^2]^{3/2}}, \qquad k \doteq \frac{1}{4\pi\epsilon_0}$$

(b) Evaluate the integral analytically in the limit $a = 0$ for arbitrary positions of the field point y_0. This represents the field generated by an infinitely long, straight line.
(c) Evaluate the integral analytically for arbitrary $a > 0$ and the field point at the focus, $y_0 = 1/4a$, of the parabola.
(d) Evaluate the integral numerically for $a = 0.5, 1, 2$ and a large range of field points. Investigate the (power-law) decay law for large positive and negative values of y_0.
(e) Plot the results for E_y vs y_0 in such ways that all salient features are demonstrated.

Solution: