

### [lex8] Electric field of parabolic line charge

Place a parabolic line charge of infinite length and with uniform line charge density  $\lambda$  into the  $xy$ -plane such that  $y = ax^2$  with  $a > 0$ .

(a) Show that the electric field at position  $y_0$  on the  $y$ -axis is given by the integral expression,

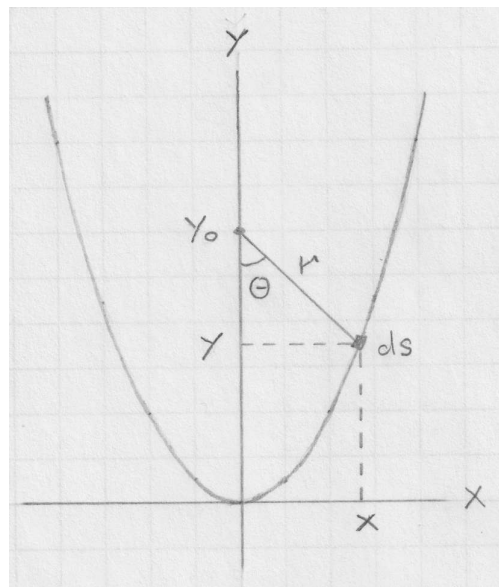
$$E_y = 2k\lambda \int_0^\infty dx \frac{\sqrt{1 + 4a^2x^2}(y_0 - ax^2)}{[x^2 + (y_0 - ax^2)^2]^{3/2}}, \quad k \doteq \frac{1}{4\pi\epsilon_0}$$

(b) Evaluate the integral analytically in the limit  $a = 0$  for arbitrary positions of the field point  $y_0$ . This represents the field generated by an infinitely long, straight line.

(c) Evaluate the integral analytically for arbitrary  $a > 0$  and the field point at the focus,  $y_0 = 1/4a$ , of the parabola.

(d) Evaluate the integral numerically for  $a = 0.5, 1, 2$  and a large range of field points. Investigate the (power-law) decay law for large positive and negative values of  $y_0$ .

(e) Plot the results for  $E_y$  vs  $y_0$  in such ways that all salient features are demonstrated.



**Solution:**