[lex74] Energy conversion in Faraday disk generator

The Faraday disk generator shown, spinning at angular velocity ω in a uniform magnetic field B, experiences, as reasoned in [lln14], an attenuating torque of magnitude,

$$N = \frac{1}{2} I B a^2, \quad I = \frac{\omega B a^2}{2R},$$

where a is the radius of the disk and R its resistance. The current I, generated by motional EMF, flows radially outward inside the disk. The loop is closed along a wire from rim to axle via sliding contacts. The acting torque slows the motion as rotational kinetic energy is converted into electrical energy (by motional EMF) and dissipated into thermal energy (by resistance).

(a) Use Newton's second law to calculate the time-dependent angular velocity $\omega(t)$ if its initial value is ω_0. Infer the time-dependent current $I(t)$ flowing through the disk.

(b) Show (by inspection) that the total energy dissipated in the current loop,

$$E_{\rm dis} \doteq \int_0^\infty dt\, R I^2(t),$$

is equal to the initial rotational kinetic energy $E_{\rm kin}^{(0)}$ of the disk as dictated by the law of energy conservation. The disk is a solid cylinder of mass m and radius a rotating about its axis.

Solution: