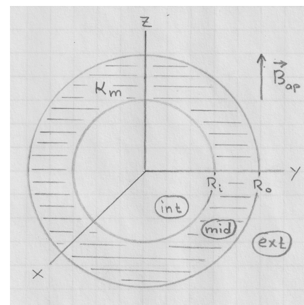
[lex73] Magnetic shielding inside a magnetizable spherical shell

A solid spherical shell of inner radius R_i and outer radius R_o, made of magnetic material with relative permeability κ_m, is placed in a region of uniform applied magnetic field $\mathbf{B}_\mathrm{ap} = B_0\,\hat{\mathbf{k}}$. We use spherical coordinates with azimuthal symmetry. The irrotational nature of the magnetic field in this situation can be described by the scalar potential $\Phi_\mathrm{m}(\mathbf{x})$. A solution of the Laplace equation, $\nabla^2\Phi_\mathrm{m} = 0$, that satisfies the symmetry in place and the boundary conditions at $r = 0$ and $r \to \infty$, as worked out in [lex72], must be of the form,

$$\Phi_\mathrm{m}^{(\mathrm{int})}(r,\theta) = ar\cos\theta, \quad \Phi_\mathrm{m}^{(\mathrm{mid})}(r,\theta) = cr\cos\theta + \frac{d\cos\theta}{r^2}, \quad \Phi_\mathrm{m}^{(\mathrm{ext})}(r,\theta) = -\frac{B_0}{\mu_0}r\cos\theta + \frac{b\cos\theta}{r^2}.$$

Determine the constants a, b, c, d by imposing relevant boundary conditions at the two interfaces. The emerging coefficient a then characterizes the shielding of the interior region from the applied external magnetic field. Determine the ratio B_int/B_0 as a function of $\kappa_\mathrm{m}, R_\mathrm{i}, R_\mathrm{o}$.



Solution: