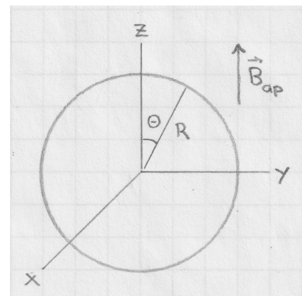
[lex72] Solid sphere placed in a uniform magnetic field

A solid sphere of radius R made of magnetic material with relative permeability κ_m is placed in a region of uniform applied magnetic field $\mathbf{B}_{\text{ap}} = B_0 \hat{\mathbf{k}}$. We use spherical coordinates with azimuthal symmetry. The irrotational nature of the magnetic field in this situation can be described by the scalar potential $\Phi_{\text{m}}(\mathbf{x})$. Solutions of the Laplace equation, $\nabla^2\Phi_{\text{m}} = 0$, that satisfy the symmetry in place are of the form,

$$\Phi_{\text{m}}^{(\text{int})}(r,\theta) = ar\cos\theta + \frac{b\cos\theta}{r^2}, \quad \Phi_{\text{m}}^{(\text{ext})}(r,\theta) = cr\cos\theta + \frac{d\cos\theta}{r^2},$$

as previously used in [lex30] for a dielectric sphere in an electic field.
Determine the constants a, b, c, d by imposing all relevant boundary conditions. Then determine the fields **H** and **B** in the interior and exterior regions. Work out the limits $\kappa_{\text{m}} \to 1$ (non-magnetizable material) and $\kappa_{\text{m}} \to 0$ (perfect diamagnet).



Solution: