[lex68] Magnetic field of uniformly magnetized sphere

Start from the result,

$$\mathbf{A}_{\rm int}(r,\theta) = \frac{\mu_0 M_0}{3}\, r\sin\theta\,\hat{\boldsymbol{\phi}}, \quad \mathbf{A}_{\rm ext}(r,\theta) = \frac{\mu_0 M_0}{3}\frac{a^3}{r^2}\,\sin\theta\,\hat{\boldsymbol{\phi}},$$

for the vector potential inside and outside a uniformly magnetized sphere of radius a as calculated in [lex67].
(a) Infer expressions $\mathbf{B}_{\rm int}(r,\theta)$ and $\mathbf{B}_{\rm ext}(r,\theta)$ in the two regions by applying the curl in spherical coordinates to both expressions.
(b) Confirm that the $\mathbf{B}_{\rm int}(r,\theta)$ is uniform and $\mathbf{B}_{\rm ext}(r,\theta)$ the field of a magnetic dipole.
(c) Confirm by inspection that these expressions are consistent with established boundary conditions for the normal field $\mathbf{B}_\perp$ and the tangential field $\mathbf{B}_\parallel$ on the surface of the sphere.

Solution: