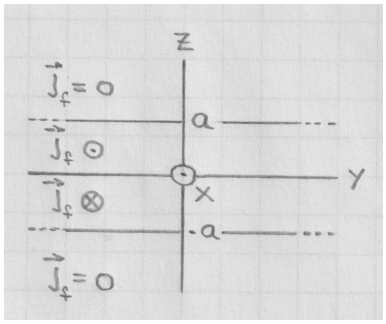
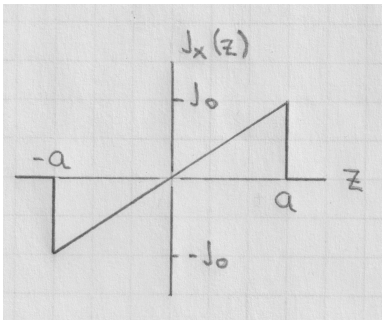
[lex66] Current-carrying magnetic slab

A slab of conducting and magnetic material of infinite lateral extensions (in x and y directions) and width $2a$ is centered at the xy-plane as shown in cross section. The magnetic suceptibility is $\chi_{\rm m}$ and the density of free current is

$$\mathbf{J}_{\rm f} = J_0 \frac{z}{a} \hat{\mathbf{i}} \quad : \ |z| \leq a.$$

(a) Use the magnetostatic relations $\nabla \cdot \mathbf{H} = 0$ and $\nabla \times \mathbf{H} = \mathbf{J}_{\rm f}$ for the magnetic field plus simplifications due to symmetry to determine the nine partial derivatives of H_x, H_y, H_z with respect to x, y, z.
(b) Establish a chain of sound reasoning which concludes that $\mathbf{H}$ vanishes identically at $|z| > a$.
(c) Determine $\mathbf{H}$ at $|z| < a$ via integration of the result from part (a).
(d) Determine the bound bulk current density $\mathbf{J}_{\rm b}$ inside the slab and the bound surface current density $\mathbf{K}_{\rm b}$ at $z = \pm a$.





Solution: