[lex62] Velocity selector

A particle with mass m and charge q traveling with velocity $\mathbf{v}$ enters a region of crossed electric and magnetic fields as shown. There it experiences an electric force $\mathbf{F}_E = q\mathbf{E}$ and a magnetic force $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$. The path of the particle stays in the xy-plane. Start from the coupled linear ODEs for the velocity components,

$$\frac{dv_x}{dt} = \frac{qB}{m} v_y, \quad \frac{dv_y}{dt} = \frac{qE}{m} - \frac{qB}{m} v_x, \tag{1}$$

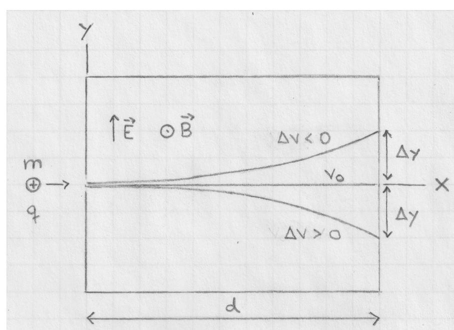
inferred in [lex50] from the equation of motion, $\mathbf{F}_E + \mathbf{F}_B = m d\mathbf{v}/dt$.
(a) Confirm that Eqs. (1) are satisfied by the trial functions,

$$v_x(t) = a\cos(\omega t) + b\sin(\omega t) + \frac{E}{B}, \quad v_y(t) = b\cos(\omega t) - a\sin(\omega t), \tag{2}$$

for $\omega = qB/m$ (cyclotron frequency) and arbitrary amplitudes a, b.
(b) Identify the values of a, b that produce a straight horizontal path for the particle traveling with constant velocity $\mathbf{v} = v_0\,\hat{\mathbf{i}}$. What is the value of v_0?
(c) Consider a particle launched with speed $v_0 + \Delta v$, where Δv can be positive or negative. Identify the values of a, b for this case.
(d) Integrate the solutions (2) for a, b as determined in (c) to produce the components $x(t), y(t)$ of the instantaneous position of the particle.
(e) Determine the lateral deviation Δy of the path at $x = d$. Assume that that $\Delta/v_0 \ll 1$ and $\omega d \ll 1$ for this part, which simplifies the analysis and should produce the result,

$$\frac{\Delta y}{d} = -\frac{\Delta v}{v_0}\frac{\omega d}{v_0}. \tag{3}$$

This setup can be used extract from a beam of particles with a broad distribution of speed one that has a very narrow distribution if we block all particles from moving past $x = d$ except those whose paths deviate from a straight line by less than $\pm\Delta y$.



Solution: