[lex61] RC circuit I: transfer of energy between devices

Consider the RC circuit shown. The battery supplies the voltage \mathcal{E} when connected to the circuit. Both capacitors have capacitance C. Both resistors have resistance R. In [lex58] we have determined the instantaneous charges $Q_1(t), Q_2(t)$ and currents $I_1(t), I_2(t)$ during three processes initiated by the following switch settings:

(a) S_1 is closed while S_2 remains open;

- (b) S_1 is opened and the S_2 is closed;
- (c) S_1 is closed while S_2 remains closed.

Calculate for each process the energy supplied by the battery via the integral,

$$U_{\rm sup} = \int_0^\infty dt \, \mathcal{E}[I_1(t) + I_2(t)],$$

the energy stored in the capacitors,

$$U_{\rm sto} = \lim_{t \to \infty} \left[\frac{Q_1(t)^2}{2C} + \frac{Q_2(t)^2}{2C} \right],$$

and the energy dissipated in the resistors,

$$U_{\rm dis} = \int_0^\infty dt \Big\{ R[I_1(t) + I_2(t)]^2 + R[I_2(t)]^2 \Big\}.$$

Confirm that all transferred energy is accounted for in the three processes. Note that none of these transferred energies depends on the resistance of the resistors. the resistances merely control the speed at which the transfer take place.



Solution: