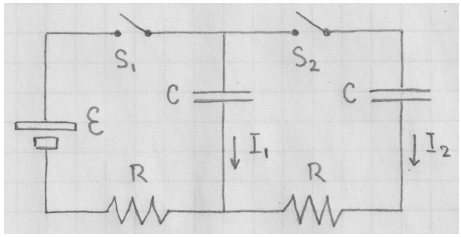
[lex58] RC circuit I with currents flowing

Consider the RC circuit shown. The battery supplies the voltage $\mathcal{E}$ when connected to the circuit. Both capacitors have capacitance C. Both resistors have resistance R. The instantaneous charges on the capacitors left and right are $Q_1(t)$ and $Q_2(t)$, respectively. The currents flowing through them are $I_1(t) = dQ_1/dt$ and $I_2(t) = dQ_2/dt$, respectively, in the directions shown. Initially, both switches S_1 and S_2 open and there is no charge on either capacitor.
(a) Use the loop rule to show that (with S_1 and S_2 closed) the instantaneous charges on the capacitors are determined by the following coupled linear ODEs:

$$\dot{Q}_1 + \dot{Q}_2 = \frac{\mathcal{E}}{R} - \frac{Q_1}{RC}, \quad \dot{Q}_2 = \frac{Q_1 - Q_2}{RC}. \tag{1}$$

(b) Simplify the first Eq. (1) with S_1 closed and S_2 open. Solve it with initial conditions $Q_1(0) = 0$.
(c) Simplify the second Eq. (1) with S_1 open and S_2 closed. Solve it for the initial state representing the long-time asymptotic equilibrium found in (b).
(d) Solve Eqs. (1) for the case when S_1 and S_2 are closed with the initial state representing the long-time asymptotic equilibrium found in (c).
(e) Plot the charges $Q_1(t)$, $Q_2(t)$ and the currents $I_1(t)$, $I_2(t)$ for the solution found in (d). Set $\mathcal{E} = 1$, $C = 1$, and $R = 1$ for the purpose of graphical representation, which amounts to using scaled variables.



Solution: