[lex51] Magnetic field of straight current segment I

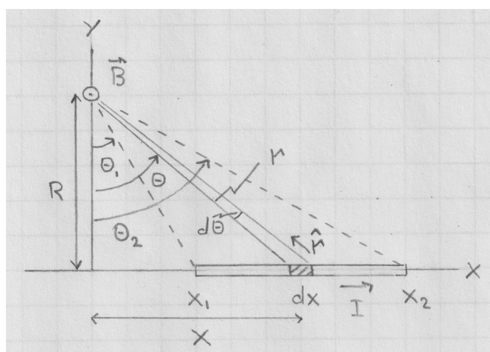
Consider a current-carrying wire that includes a straight segment. The goal here is to calculate the magnetic field $\mathbf{B}$ generated by that segment at an arbitrary field point. Without loss of generality we can choose a Cartesian coordinate system such that the segment is on the x-axis with current I flowing in the positive direction and the field point positioned on the positive y-axis as shown. The magnetic field is then directed in positive z-direction (out of the plane), dictated by the cross-product $\hat{\mathbf{i}} \times \hat{\mathbf{r}}$ between current direction and distance vector from source point to field point. Show that the magnitude of the field is given by the expression,

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}(\sin\theta_2 - \sin\theta_1), \qquad L = R(\tan\theta_2 - \tan\theta_1),$$

where L is the length of the segment. The angles θ_1, θ_2 are from the vertical to the lines connecting the rear end and the front end of the segment, respectively, with the field point. The y-coordinate of the field point is $R > 0$. If the segment extends over the entire x-axis, we have $\theta_1 = -\pi/2$ and $\theta_2 = +\pi/2$, which which recovers the familiar expression,

$$B = \frac{\mu_0 I}{2\pi R}.$$

Hint: Start from the Biot-Savart law and perform a variable transformation from x to θ, which simplifies the integral.



Solution: