## [lex50] Motion in crossed electric and magnetic fields

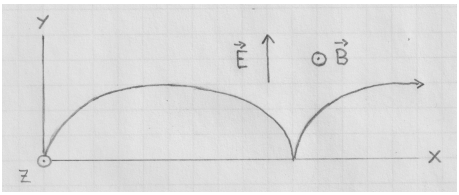
In a region of uniform electric field $\mathbf{E} = E\,\hat{\mathbf{j}}$ and uniform magnetic field $\mathbf{B} = B\,\hat{\mathbf{k}}$ a particle of mass $m$ and charge $q$ is released from rest at $\mathbf{x} = 0$.
(a) Express Newton's second law, $\mathbf{F} = m\mathbf{a}$, with the Lorentz force, $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, as a set of three $1^{\text{st}}$-order ODEs for the velocity components $v_x, v_y, v_z$.
(b) Show that these coupled linear ODEs are satisfied by the trial functions,

$$v_i(t) = a_i \cos(\omega t) + b_i \sin(\omega t) + c_i, \quad i = x, y, z,$$

if the following conditions are imposed on the parameters $a_i, b_i, \omega$:

$$\omega = \frac{qB}{m}, \quad a_x = -b_y, \quad a_y = b_x, \quad c_x = \frac{E}{B}, \quad c_y = 0, \quad a_z = b_z = 0.$$

(c) Impose the initial condition, $\mathbf{v}(0) = 0$ to fully determine all parameters and write explicit expressions for $v_x(t), v_y(t), v_z(t)$.
(d) Integrate these expressions to infer explicit expressions for $x(t), y(t), z(t)$ using the initial condition $\mathbf{x}(0) = 0$. The path of the particle turns out to be a cycloid in the $xy$-plane.




**Solution:**