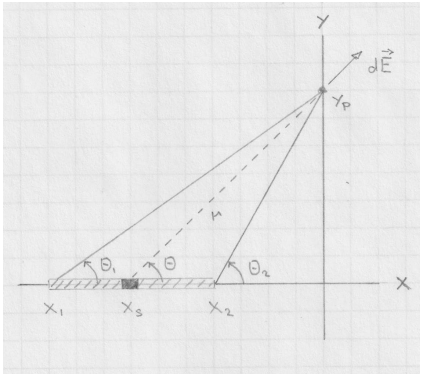
[lex5] Electric field of a charged rod II

Here we extend the calculation of [lex1] and calculate the electric field of a uniformly charged rod with length L and line charge density λ at a field point in arbitrary position relative to the rod. Without loss of generality we can place the rod on the x-axis and the field point on the positive y-axis of a Cartesian coordinate system. Show that the two non-vanishing components of the electric field are given by the expressions,

$$E_x = \frac{k\lambda}{y_P}\big(\sin\theta_2 - \sin\theta_1\big), \quad E_y = -\frac{k\lambda}{y_P}\big(\cos\theta_2 - \cos\theta_1\big), \quad k \doteq \frac{1}{4\pi\epsilon_0},$$

where θ_1, θ_2 represent the angles above the horizontal of the lines connecting the left and right ends of the rod, respectively, with the field point. The vertical coordinate of the field point is $y_P > 0$. Note that the range of both angles is $0 \leq \theta_i \leq \pi$. If the rod extends over the entire x-axis, we have $\theta_1 = 0$ and $\theta_2 = \pi$.



Solution: