[lex41] Electric flux through cube and net charge inside

Consider a cube with sides of length 2m positioned and oriented in a coordinate system as shown in the presence of an electric field,

$$\mathbf{E}(\mathbf{x}) = a\,\hat{\mathbf{i}} + by\,\hat{\mathbf{j}} + cz^2\,\hat{\mathbf{k}}; \quad a = 4\mathrm{V/m}, \quad b = 3\mathrm{V/m}^2, \quad c = 2\mathrm{V/m}^3.$$

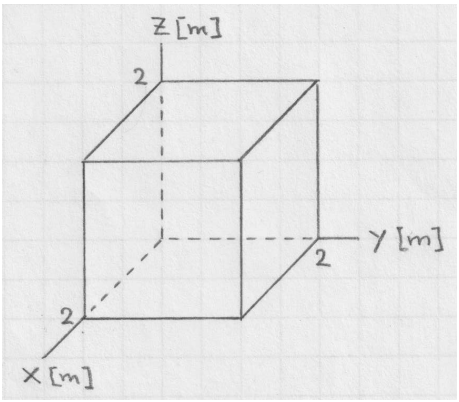
(a) Show that $\mathbf{E}$ is irrotational, thus qualifies as an electrostatic field.
(b) Calculate the net charge Q_{in} inside the cube via the integral version of Gauss's law,

$$\oint_S d\mathbf{a} \cdot \mathbf{E} = \frac{Q_{\mathrm{in}}}{\epsilon_0}, \quad \epsilon_0 = 8.85 \times 10^{-12}\mathrm{C}^2\mathrm{N}^{-1}\mathrm{m}^{-2},$$

from the electric flux Φ_E through the surface of the cube (integral over closed surface).
(c) Calculate Q_{in} from the differential version of Gauss's law,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0},$$

as a volume integral of the charge density ρ inside the cube.



Solution: