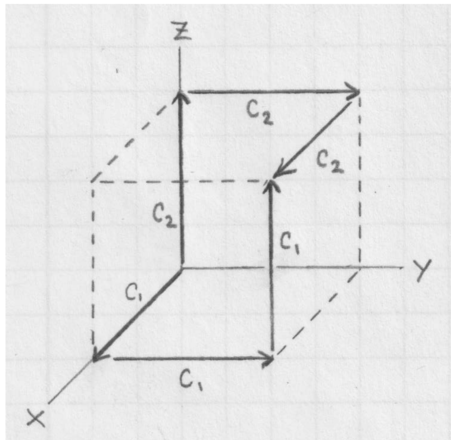
[lex40] From electric potential to electric field and back

(a) From the electric potential, $\Phi(\mathbf{x}) = ax^3 + byz^2 - cxyz$, where a, b, c are constants in units of $\mathrm{V/m^3}$, determine the electric field in the form $\mathbf{E}(\mathbf{x}) = E_x(x,y,z)\,\hat{\mathbf{i}} + E_y(x,y,z)\,\hat{\mathbf{j}} + E_z(x,y,z)\,\hat{\mathbf{k}}$.
(b) Show that electric field, $\mathbf{E}(\mathbf{x}) = -2axz\,\hat{\mathbf{i}} + 3by^2\,\hat{\mathbf{j}} - ax^2\,\hat{\mathbf{k}}$ is irrotational, i.e. satisfies $\nabla \times \mathbf{E} = 0$, and thus qualifies as an electrostatic field.
(c) Infer from this electrostatic field the electric potential $\Phi(x,y,z)$ via integration along path C_1 and along path C_2 as shown. The irrotational nature of the electric field guarantees that the integral is path-independent.



Solution: