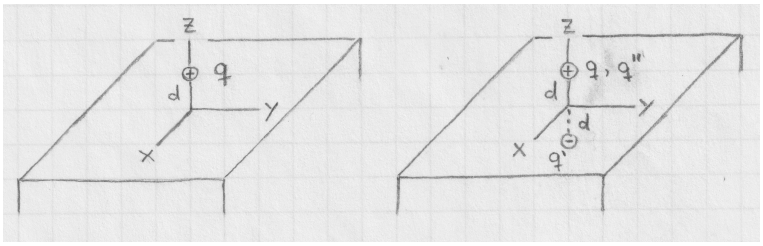
[lex32] Point charge near plane surface of dielectric I

A point charge $q > 0$ is positioned at $z = d > 0$ above the plane surface of a uniform dielectric with dielectric constant $\kappa > 1$. The polarization of the dielectric material modifies the electric field generated by the point charge at $z < 0$ inside the dielectric. The bound surface charge density associated with the polarization, in turn, modifies the electric field at $z > 0$ outside the dielectric. The boundary conditions, $\Delta \mathbf{E}_{\parallel} = 0$ and $\Delta \mathbf{D}_{\perp} = 0$, as established in [lln9], can be satisfied if we introduce two (virtual) image charges, q' at $z = -d$ to adjust the electric field at $z > 0$, and q'' at $z = d$ to generate the electric field at $z < 0$.
(a) Show that the image charges are

$$q' = \frac{1-\kappa}{\kappa+1}\, q, \qquad q'' = \frac{2}{\kappa+1}\, q.$$

(b) Make contact with known results for the case of no dielectric, ($\kappa = 1$), and the limiting case of a conducting material ($\kappa \to \infty$).

Hint: Start with expressions for the electric potential and then derive the relevant field components from those expressions.



Solution: