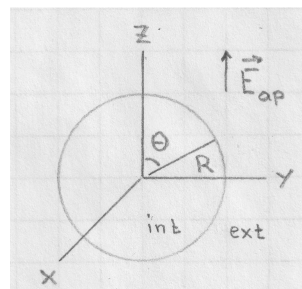
[lex30] Dielectric sphere polarized by uniform electric field

A uniformly dielectric solid sphere of radius R and dielectric constant κ is placed into a region of uniform applied electric field $\mathbf{E}_{\rm ap} = E_0\,\hat{\mathbf{k}}$. We use spherical coordinates r, θ with azimuthal symmetry. The polarization of the dielectric material produces bound charge at the surface, which modifies the electric field both outside and inside the sphere. Start from a special case of the model solution of the Laplace equation established in [lln6]:

$$\Phi_{\rm int}(r,\theta) = ar\cos\theta + \frac{b\cos\theta}{r^2}, \quad \Phi_{\rm ext}(r,\theta) = cr\cos\theta + \frac{d\cos\theta}{r^2}.$$

(a) Establish the functions $\Phi_{\rm int}(r,\theta)$ $\Phi_{\rm ext}(r,\theta)$ by determining the constants a, b, c, d by imposing all relevant boundary conditions.
(b) Check the limits $\kappa \to 1$ (non-polarizable material) and $\kappa \to \infty$ (conducting material). The latter case was worked out in [lex17].
(c) Determine the surface density of bound charge, $\sigma_{\rm b}(\theta)$, which, in the limit $\kappa \to \infty$ must approach the surface density of mobile charge on a conducting sphere calculated in [lex17].



Solution: