[lex29] Uniformly polarized dielectric sphere

A dielectric sphere of radius R which is uniformly polarized generates an electric field both inside and outside as well as a density of bound charge on its surface. Here we disregard the agent that causes the polarization, which is an external electric field of some form and strength. We use spherical coordinates with azimuthal symmetry. The (uniform) polarization is

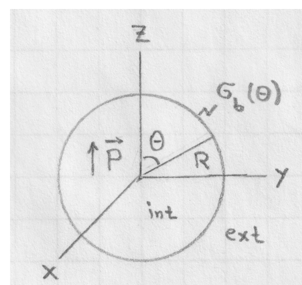
$$\mathbf{P}(\mathbf{x}) = P_0\,\hat{\mathbf{k}} \quad : \ |\mathbf{x}| < R. \tag{1}$$

(a) Determine the surface charge density $\sigma_{\rm b}(\theta)$ of bound charge on the sphere.
(b) Solve the Laplace equation with Neumann boundary conditions to infer the electric potential $\Phi_{\rm int}(r,\theta)$ and $\Phi_{\rm ext}(r,\theta)$ in the interior and exterior regions, respectively. The imposition of boundary conditions and continuity at the surface of the sphere leaves one undetermined parameter in the expressions.
(c) Infer the components $E_r^{\rm int}$, $E_\theta^{\rm int}$ of the interior electric field and the components $E_r^{\rm ext}$, $E_\theta^{\rm ext}$ of the exterior electric field via gradient of the potential.
(d) Use the relation between surface charge density and normal electric field determine the aformentioned parameter.
(e) Show that the electric field inside is uniform and the electric potential outside is that of an electric dipole with dipole moment $\mathbf{p}$. Determine the dipole moment $\mathbf{p}$ in relation to the polarization $\mathbf{P}$.

Hint: Start from the model solution,

$$\Phi(r,\theta) = \frac{a}{r} + b + \frac{c\cos\theta}{r^2} + dr\cos\theta, \tag{2}$$

of the Laplace equation determined in [lln6] for arbitrary constants a, b, c, d.



Solution: