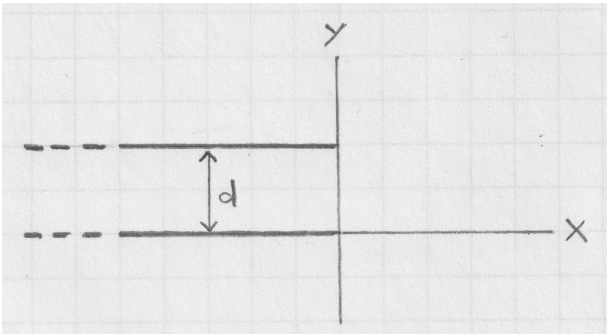
[lex28] Fringe electric potential and fringe field of parallel plates

The electric field between the plates of a parallel-plate capacitor is uniform and perpendicular to the plane of the plates. The electric potential varies linearly with the position coordinate in field direction. This is close to exact except near the edges. Here we investigate the deviations from this simple behavior at one edge. We assume that the parallel plates are horizontal as shown, one plate at $y = 0$ and the other at $y = d > 0$. The edge is at $x = 0$. Use the method of conjugate functions from [lln7] to find the electric potential $\Phi(x, y)$. As it turns out, the analytic function,

$$F(z) \doteq \frac{\Phi_0}{\imath\pi} \ln w, \quad z = \frac{d}{\pi}\left[\ln w + \frac{1}{2}(1 - w^2)\right] = x + \imath y. \tag{1}$$

does the trick. Φ_0 is the potential of the plate at $y = d$ whereas the plate at $y = 0$ is grounded. The functional relation between the complex variables w and z is an instance of conformal mapping.
(a) Identify the conjugate functions $g(x, y)$ and $h(x, y)$ in a parametric representation, $x(\rho, \theta), y(\rho, \theta)$, using polar coordinates for w: $w = \rho e^{\imath\theta}$.
(b) Produce a graphical representation of sets of equipotential lines and sets of field lines lines in a way that demonstrates all salient features of fringe potential and fringe electric fields.



Solution: