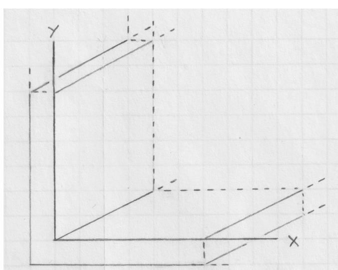
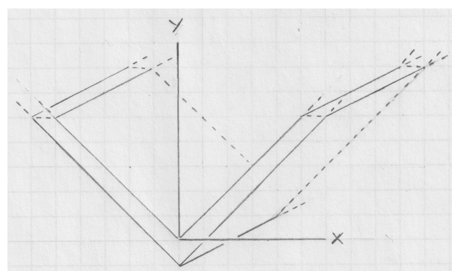
[lex26] Conducting plates intersecting at right angle

A charged conducting plate of nonzero width and infinite extension is positioned in the horizontal plane. The surface charge density σ is uniform. The electric field generated on either side of the plate is uniform and directed vertically.
When half of the plate is bent by 90° into vertical direction as shown on the left, a new equilibrium with nonuniform charge distribution and nonuniform electric field will be established.
For the analysis of this situation we seek a solution of the Laplace equation at $x > 0$ and $y > 0$ with the boundary condition, $\Phi(x, 0) = \Phi(0, y) = 0$ by employing the method of conjugate functions from [lln7]. The complex analytic function which does the trick in this case reads,

$$F(z) \doteq Az^2 = g(x, y) + \imath h(x, y), \quad z \doteq x + \imath y.$$

(a) Identify the real part $g(x, y)$ and the imaginary part $h(x, y)$ and show that they satisfy the Cauchy-Riemann conditions, which makes them conjugate functions.
(b) Assign the equipotential lines and the field lines to $g(x, y) = \text{const}$ and $h(x, y) = \text{const}$ in such a way that the aforementioned boundary conditions are satisfied.
(c) Design graphical representations of potential and field as sets of lines that intersect orthogonally.
(d) From the gradient of the function $\Phi(x, y)$ evaluated at the surface of the conductor, derive the surface charge density using the local relation $E = \sigma/\epsilon$.
(e) Show that if we switch the assignments of equipotential lines and field lines, we describe the solution of the Laplace equation for the configuration shown on the right.





Solution: