# [lex25] Conducting half-cylindrical shells at different electric potential

Consider a thin and long conducting cylindrical shell of radius $R$ split into two halves with semicircular cross section, electrically insulated from each other. We use cylindrical coordinates $(r, \phi, z)$ and translational invariance along the $z$-axis. The half at $-\pi/2 < \phi < \pi/2$ ($\pi/2 < \phi < 3\pi/2$) is at electric potential $+\Phi_0$ ($-\Phi_0$). These are Dirichlet boundary conditions. From [lln7] we know that the electric potential in the interior and exterior regions can be expanded in the form,

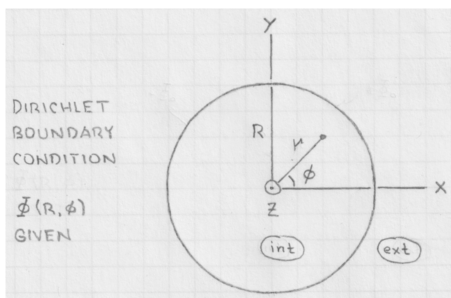
$$\Phi_{\text{int}}(r,\phi) = \sum_{n=1}^{\infty} c_n \left(\frac{r}{R}\right)^n \cos(n\phi) \quad : \ r \leq R, \tag{1}$$

$$\Phi_{\text{ext}}(r,\phi) = \sum_{n=1}^{\infty} c_n \left(\frac{R}{r}\right)^n \cos(n\phi) \quad : \ r \geq R, \tag{2}$$

where we have already used the reflection symmetry about the $x$-axis.
(a) Show that the expansion coefficients $c_n$, derived as explained in [lln7], are

$$c_n = \frac{4\Phi_0}{n\pi} \sin\left(\frac{n\pi}{2}\right).$$

(b) Find the surface charge density $\sigma_0(\phi)$ on each half as expansions derived from (1) and (2).
(c) Design graphical representations for the functions (1) and (2) that show all salient features. Then describe those features. Plot and interpret the function $\sigma_0(\phi)$ obtained in part (b).




**Solution:**