[lex23] Oppositely charged hemispherical shells

Consider a thin spherical shell of radius R split with oppositely charged hemispheres. We analyze the two cases with the following azimuthally symmetric surface charge densities:

$$\text{(i)}\quad \sigma(\theta) = \sigma_0 \cos\theta, \qquad \text{(ii)}\quad \sigma(\theta) = \sigma_0 \mathrm{sgn}\big(\cos\theta\big).$$

These are Neumann boundary conditions. From [lln7] we know that the electric potential in the interior and exterior regions can be expanded in the form,

$$\Phi_{\text{int}}(r,\theta) = \sum_{l=0}^{\infty} a_l \left(\frac{r}{R}\right)^l P_l(\cos\theta) \quad : \ r \leq R, \tag{1}$$

$$\Phi_{\text{ext}}(r,\theta) = \sum_{l=0}^{\infty} a_l \left(\frac{R}{r}\right)^{l+1} P_l(\cos\theta) \quad : \ r \geq R, \tag{2}$$

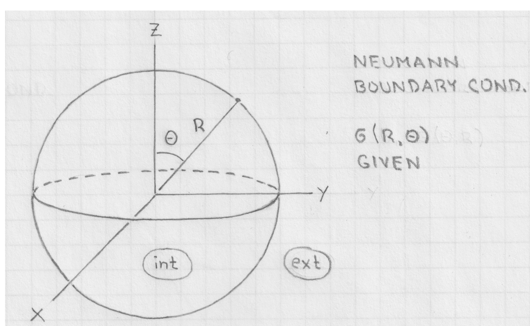
where the $P_l(\cos\theta)$ are Legendre polynomials and r, θ are the radial and polar components of spherical coordinates. We also know from [lln7] how the expansion coefficients a_l can be derived from the surface charge density $\sigma(\theta)$:

$$a_l = \frac{R}{2\epsilon_0} \int_0^{\pi} d\theta \sin\theta \, P_l(\cos\theta)\sigma(\theta). \tag{3}$$

(a) Show that the expansion coefficients a_l pertaining to this application are,

$$\text{(i)}\quad a_l = \frac{R\sigma_0}{3\epsilon_0}\,\delta_{l,1} \qquad \text{(ii)}\quad a_l = \frac{R\sigma_0}{(2l+1)\epsilon_0}\big[P_{l-1}(0) - P_{l+1}(0)\big].$$

Restate the simplified expressions for electric potential in the interior and exterior regions. Analyze the two cases (i) and (ii) separately.
(b) Design graphical representations for the functions (1) and (2) that show all salient features, including the main difference of the two cases.



Solution: Mathematica lex23.nb