[lex22] Conducting hemispherical shells at different electric potential

Consider a thin conducting spherical shell of radius R split into two hemispheres and electrically insulated from each other. The hemisphere at $z > 0$ ($z < 0$) is at electric potential $+\Phi_0$ ($-\Phi_0$). These are Dirichlet boundary conditions. From [lln7] we know that the electric potential in the interior and exterior regions can be expanded in the form,

$$\Phi_{\text{int}}(r, \theta) = \sum_{l=0}^{\infty} a_l \left(\frac{r}{R}\right)^l P_l(\cos\theta) \quad : \ r \leq R, \tag{1}$$

$$\Phi_{\text{ext}}(r, \theta) = \sum_{l=0}^{\infty} a_l \left(\frac{R}{r}\right)^{l+1} P_l(\cos\theta) \quad : \ r \geq R, \tag{2}$$

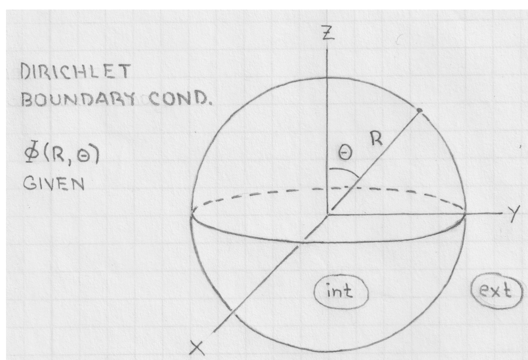
where the $P_l(\cos\theta)$ are Legendre polynomials and r, θ are the radial and polar components of spherical coordinates. We also know from [lln7] how the expansion coefficients a_l can be derived from the given potential $\Phi(R, \theta)$:

$$\Rightarrow \ a_l = \frac{2l+1}{2} \int_0^{\pi} d\theta \sin\theta \, P_l(\cos\theta) \Phi(R, \theta), \tag{3}$$

(a) Show that the expansion coefficients a_l pertaining to this application are,

$$a_l = \Phi_0 \big[P_{l-1}(0) - P_{l+1}(0)\big].$$

(b) Design graphical representations for the functions (1) and (2) that show all salient features.
(c) Determine the surface charge density $\sigma(\theta)$ on each hemisphere as an expansion in terms of Legendre polynomials. Plot and interpret the function $\sigma(\theta)$ thus obtained.



Solution: