

[lex21] **Conducting rectangular pipe with end-plate potential**

Consider a grounded conducting pipe at $x > 0$ with an inside square cross section $0 \leq y \leq a$, $0 \leq z \leq a$ and an end-plate at $x = 0$ held at potential $\Phi_0 > 0$.

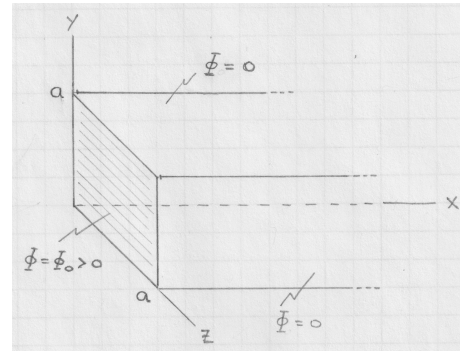
(a) Express the electric potential $\Phi(x, y, z)$ in the interior of the pipe as a series of product functions $X(x)Y(y)Z(z)$ that solve the Laplace equation and satisfy the Dirichlet boundary conditions. Explain why that expansion must have the form,

$$\Phi(x, y, z) = C_{mn} e^{-\kappa_{mn} x} \sin(k_m y) \sin(k_n z); \quad k_m = \frac{m\pi}{a}, \quad k_n = \frac{n\pi}{a}, \quad \kappa_{mn} = \sqrt{k_m^2 + k_n^2},$$

and determine the permissible integers n, m .

(b) Visualize the potential graphically in ways that emphasize all salient features.

(c) Determine and visualize the surface charge distribution in each wall and in the endplate.



Solution: