

[lex20] Square pipe made of conducting walls at different potential

Consider a pipe of infinite length with an $a \times a$ inside square cross section, made of four conducting walls kept at different electric potential. The walls at $x = \pm a/2$ are grounded and the walls at $y = \pm a/2$ are held at potential $\Phi_0 > 0$.

(a) Express the electric potential $\Phi(x, y)$ inside the pipe as a series of product functions $X(x)Y(y)$ that solve the Laplace equation and satisfy the Dirichlet boundary conditions. Explain why that series must have the form,

$$\Phi(x, y) = \sum_{n=0}^{\infty} C_n \cos(k_n x) \cosh(k_n y), \quad k_n = (2n + 1) \frac{\pi}{a} \quad : \quad n = 0, 1, 2, \dots$$

(b) Visualize the potential in a contour plot.

(c) Determine the surface charge distribution in each wall and plot its profile.

Solution:

