[lex196] Magnetic dipole near long straight current-carrying wire

An infinitely long thin wire is positioned on the z-axis and has a steady current $I > 0$ flowing in the negative z-direction. A magnetic dipole with dipole moment,

$$\mathbf{m} = m_x\,\hat{\mathbf{i}} + m_y\,\hat{\mathbf{j}} + m_z\,\hat{\mathbf{k}}, \quad m_x = m\sin\theta\cos\phi, \quad m_y = m\sin\theta\sin\phi, \quad m_z = m\cos\theta,$$

is positioned at $x = 0$, $y > 0$, $z > 0$. In the magnetic field $\mathbf{B}$ of the current, the dipole has potential energy $U = -\mathbf{m}\cdot\mathbf{B}$, experiences a torque $\mathbf{N} = \mathbf{m}\times\mathbf{B}$, and a force $\mathbf{F} = -\nabla U$.
(a) For which orientation θ, ϕ of $\mathbf{m}$ does U have its minimum value.
(b) Find the torque $\mathbf{N}$ if the dipole moment $\mathbf{m}$ is oriented in the positive z-direction.
(c) Find the force $\mathbf{F}$ acting on the dipole if its moment is oriented (i) in positive z-direction, (ii) in positive y-direction, (iii) in positive x-direction, (iv) in negative x-direction.

Solution: