

[lex195] Electric dipole near long electrically charged rod

An infinitely long thin rod with uniform charge density $\lambda > 0$ is positioned on the z -axis. An electric dipole with dipole moment,

$$\mathbf{p} = p_x \hat{\mathbf{i}} + p_y \hat{\mathbf{j}} + p_z \hat{\mathbf{k}}, \quad p_x = p \sin \theta \cos \phi, \quad p_y = p \sin \theta \sin \phi, \quad p_z = p \cos \theta,$$

is positioned at $x = 0$, $y > 0$, $z > 0$. In the electric field \mathbf{E} of the rod, the dipole has potential energy $U = -\mathbf{p} \cdot \mathbf{E}$, experiences a torque $\mathbf{N} = \mathbf{p} \times \mathbf{E}$, and a force $\mathbf{F} = -\nabla U$.

- (a) For which orientation θ, ϕ of \mathbf{p} does U have its minimum value.
- (b) Find the torque \mathbf{N} if the dipole moment \mathbf{p} is oriented in the positive z -direction.
- (c) Find the force \mathbf{F} acting on the dipole if its moment is oriented (i) in positive x -direction, (ii) in positive z -direction, (iii) in positive y -direction, (iv) in negative y -direction.

Solution: