

[lex194] Lorentz invariance of Maxwell's equation

A demonstration of Maxwell's equations written in the standard form,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J},$$

requires knowledge of how $E_i, B_i, \rho, J_i, \partial/\partial x_i$ for $i = x, y, z$ Lorentz transform. This information is available in [lln25]. We assume that the relative motion between inertial frames is $\mathbf{v} = v \hat{\mathbf{i}}$. The Lorentz invariance is worked out step by step in [lln25] for the the first two Maxwell equations and for the component with $\partial B_x/\partial t$ of Faraday's law.

Demonstrate the invariance of Faraday's law for the other two components and the invariance of Ampère's law for all three components using analogous steps.

Solution: