[lex193] AC circuit with RLC devices in series

Consider the RLC series circuit shown. The voltage provided by the power source and the resulting current are described by the expressions,

$$\mathcal{E}(t) = V_{\mathcal{E}} e^{\imath\omega t}, \quad I(t) = I_{\mathcal{E}} e^{\imath(\omega t - \delta_{\mathcal{E}})},$$

where $V_{\mathcal{E}}$, ω are given and $I_{\mathcal{E}}$, $\delta_{\mathcal{E}}$ are determined in [lam28]. Assume that $L/C > R^2$.
(a) Show that the voltage amplitudes across the inductor and the capacitor both assume the same value at the resonance frequency:

$$V_L^{max}(\omega_0) = V_L^{max}(\omega_0) = V_0^{max} = V_{\mathcal{E}}\sqrt{\frac{\tau_{RL}}{\tau_{RC}}}, \quad \omega_0 = \frac{1}{\sqrt{LC}},$$

where $\tau_{RC} = RC$ and $\tau_{RL} = L/R$ are relaxation times associated with RC and RL circuits.
(b) Show that both voltage amplitudes reach the same higher maximum value,

$$V_L^{max}(\omega_L) = V_C^{max}(\omega_C) = \frac{V_0^{max}}{\sqrt{1 - \tau_{RC}/4\tau_{RL}}}, \quad V_0^{max} = V_{\mathcal{E}}\sqrt{\frac{\tau_{RL}}{\tau_{RC}}},$$

at the shifted frequencies,

$$\omega_L = \frac{\omega_0}{\sqrt{1 - \tau_{RC}/2\tau_{RL}}} > \omega_0, \quad \omega_C = \omega_0\sqrt{1 - \tau_{RC}/2\tau_{RL}} < \omega_0.$$

Solution: