[lex191] Vector potential generated by magnetized material

Consider a compact region V of magnetized material characterized by a magnetization $\mathbf{M}(\mathbf{x}')$. The vector potential potential at any field point is then determined by the integral expression [lln13],

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_V d^3x' \frac{\mathbf{M}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3}.$$

Show that this expression can be transformed into

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \left[\int_V d^3x' \frac{\nabla' \times \mathbf{M}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} + \oint_S \frac{\mathbf{M}(\mathbf{x}') \times d\mathbf{a}'}{|\mathbf{x} - \mathbf{x}'|} \right],$$

where S is the surface of that compact region.

Solution: