[lex190] Current density and magnetic dipole moment

Consider a compact region in space with a current density $\mathbf{J}(\mathbf{x}')$ present. The magnetic dipole moment generated by this current density is defined as follows:

$$\mathbf{m} \doteq \frac{1}{2} \int_V d^3x' \, \mathbf{x}' \times \mathbf{J}(\mathbf{x}').$$

If $\hat{\mathbf{r}}$ is a (fixed) unit vector pointing to a distant field point, show that the relation,

$$\int_V d^3x' \, \mathbf{J}(\mathbf{x}')(\hat{\mathbf{r}} \cdot \mathbf{x}') = \mathbf{m} \times \hat{\mathbf{r}},$$

holds, which is instrumental for establishing the dipole contribution to the vector potential of a localized current distribution.

Solution: