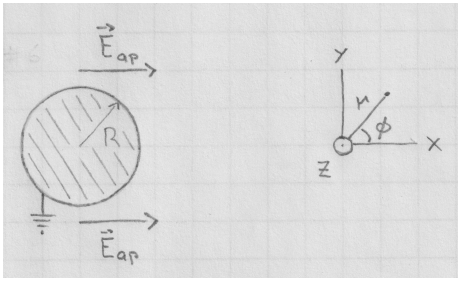
[lex19] Conducting cylinder in uniform electric field

Into a region of uniform electric field $\mathbf{E} = E_0\,\hat{\mathbf{i}}$ a grounded conducting cylinder of radius R and infinite length is placed such that its axis coincides with the z-axis of the cylindrical coordinate system in use. Electrostatic equilibrium is restored by a nonuniform surface charge density σ, which modifies the electric field surrounding the cylinder.
(a) Show that an electric potential of the form,

$$\Phi(r,\phi) = a\ln\frac{r}{b} + \frac{c\cos\phi}{r} + dr\cos\phi, \tag{1}$$

(in cylindrical coordinates) is a solution of the Laplace equation.
(b) Determine the coefficients a, b, c, d in (1) such that physically meaningful boundary conditions at $r = R$ and at $r = \infty$ are satisfied.
(c) Determine the surface charge density $\sigma(R,\phi)$.
(d) One half of the conducting surface will be positively charged and the other negatively. Find the induced charge per unit length on each side.
(e) If the grounded cylinder is replaced a conducting cylinder that carries a charge λ_0 per unit length on its surface, what are the coefficients a, b, c, d in (1) for that case?



Solution: