[lex186] Mixed versus pure photonic states

Consider the two photonic number states $|0\rangle$ and $|1\rangle$. Both are stationary states. Their Husimi distributions are time-independent.
(a) Combine the two states $|0\rangle$ and $|1\rangle$ into a mixed state,

$$\rho_{\rm m} = \frac{1}{2}\big[|0\rangle\langle 0| + |1\rangle\langle 1|\big].$$

Determine its (properly normalized) Husimi distribution, $W_{\rm m}(x,p)$ and establish a 3D plot similar to the one shown in [lln26].
(b) Combine the two states $|0\rangle$ and $|1\rangle$ into a pure quantum state,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big[|0\rangle + |1\rangle\big], \quad \rho_{\rm e} = |\psi\rangle\langle\psi|.$$

Determine its (properly normalized) Husimi distribution, $W_{\rm e}(x,p,t)$, which is time-dependent and plot it at different times to demonstrate the periodicity.
(c) Show that the the tme-average of the function $W_{\rm e}(x,p,t)$ yields the function $W_{\rm m}(x,p)$.

Solution: