

[lex183] Quantum uncertainty in ground state of harmonic oscillator

Consider the quantum harmonic oscillator as introduced in [lam5]. The task in this exercise is to calculate (for the ground state) the uncertainties in position and momentum, ΔQ and ΔP , respectively, and confirm that the product assumes the minimum value, $\frac{1}{2}\hbar$, allowed by Heisenberg's inequality introduced in [lam4].

- (a) Perform the calculation using the wave function $\psi_0(x)$ in position space.
- (b) Perform the calculation using the wave function $\tilde{\psi}(p)$ in momentum space.
- (c) Perform the calculation on the basis of the number state $|0\rangle$ (vacuum) with the position and momentum operators expressed as linear combinations of ladder operators.

Solution: