[lex180] Photon detection events from thermal radiation.

Whereas detection events of coherent light involve a random sequence of single photons as described by the Poisson statistics, the Pascal statistics of thermal radiation (of one wavelength) indicate that detection events generally involve multiple photons.
(a) Show that for the Pascal distribution [lln24],

$$P_n(t) = \frac{(t/\tau)^n}{(1+t/\tau)^{n+1}}, \quad f(t) = -\frac{dP_0}{dt} = \frac{1/\tau}{(1+t/\tau)^2},$$

the following relations hold:

$$\int_0^t dt' f(t') P_0(t-t') = \sum_{n=1}^{\infty} 2^{-n} P_n(t) + \frac{2}{(2+t/\tau)^2} \ln\left(1+\frac{t}{\tau}\right),$$

$$\int_0^t dt' f(t') P_1(t-t') = \sum_{n=2}^{\infty} (n-1) 2^{-n} P_n(t) + \frac{2t/\tau}{(2+t/\tau)^3} \ln\left(1+\frac{t}{\tau}\right).$$

Solution: